

Self-assessment: 12 Basic differentiation and its applications

1. Differentiate the following:

(a) $\sqrt{x} - \frac{1}{\sqrt{x}}$

(b) $\tan x + 2 \cos x$

(c) $x^2 - e^x$

(d) $3 \ln x - 1$

[8 marks]

2. Do not use a calculator to answer this question.

Find the equation of the normal to the curve $y = 2x - \ln x$ at the point where $x = 3$.

(accessible to students on the path to grade 3 or 4) [6 marks]

3. Find the exact coordinates of the stationary point on the graph of $y = 3e^x - x$.

(accessible to students on the path to grade 3 or 4) [6 marks]

4. (a) (i) Expand and simplify $(x + h)^2 - x^2$.

(ii) Hence prove from first principles that the derivative of x^2 is $2x$.

(accessible to students on the path to grade 5 or 6)

(b) The function f is defined by $f(x) = x^2 + 4 \cos x$ for $0 < x < \pi$.

(i) By considering the graphs of $y = x$ and $y = 2 \sin x$, show that $f(x)$ has only one stationary point, and explain why this stationary point is between $\frac{\pi}{2}$ and π .

(ii) Find $f''(x)$ and hence prove that the stationary point is a minimum.

(iii) Find the coordinates of the point of inflection on the graph of $y = f(x)$.

(iv) Sketch the graph of $y = f(x)$, clearly labelling the stationary point and the point of inflection.

(accessible to students on the path to grade 7)

[19 marks]